STABILITY OF THE COMBUSTION PROCESS IN ELASTIC COMBUSTION CHAMBERS

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High-frequency instability phenomena in rigid combustion chambers have been studied theoretically in [1–3]. This phenomenon is attributed to the interaction between the combustion processes and combustion-product fluctuations in the chamber. One of the possible mechanisms of formation of high-frequency instability is examined in [3], where the combustion rate is represented in the form of a retarded pressure functional. In this case, the problem is reduced to studying the stability of a certain distributed self-oscillating time-lag system.

If the oscillation frequencies of the combustion products are comparable to the natural vibrations of the shell which forms the combustion chamber, then it is natural to expect that the elasticity of the chamber walls will affect the combustion process. Coupled effects of acoustoelastic instability can arise, in whose development the vibrations of the chamber wall play a substantial role. These effects are particularly undesirable from the point of view of the vibrational stability of combustion chambers.

In this paper, a theory of high-frequency instability of stationary combustion is developed with allowance for elastic deformations of the combustion chamber walls. The theory is based on the mechanism of vibrational combustion [1–3], according to which the combustion front is assumed to the concentrated, while the velocity jump at the front is expressed through a retarded pressure functional. It is assumed that the combustion product flow is one-dimensional and isentropic and that the chamber is cylindrical. The deformations of the chamber are described via the moment theory of shells. The existence is revealed of additional instability regions produced by the interaction between the elastic vibrations of the chamber walls and the acoustic oscillations of the combustion products. The influence of the relation between the elastic and acoustic frequencies and of the structural damping factor in the combustion chamber walls on the stability of the stationary combustion process is examined. The problem discussed is treated as a mathematical model for more complex asymmetric problems in which the elastic and acoustic frequencies can be of the same order.

1. Let us examine the combustion process in a cylindrical chamber of radius R and length L under the following assumptions: the motion of the propellant and combustion products is one-dimensional; the combustion front is plane and is concentrated in a cross section located at a distance x_0 from the distributing nozzle; the velocity jump at the combustion front is expressed, according to [3], through a retarded pressure functional; the disturbance superposed on the stationary combustion process is small; the pressure and velocity perturbations at the nozzle inlet ($x = L$) are interrelated through the acoustic admittance of the nozzle; effects in the supply system have no influence on the process inside the chamber; the distributor head is absolutely rigid; the deformations of the chamber (assuming a one-dimensional flow) are axisymmetric and are described by the moment theory of shells; and the structural damping forces are proportional to the velocity.

The combustion product flow is described by the equation of motion, the continuity equation, and the equation of state

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x},$$
$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho u S) = 0, \qquad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\varkappa}, \tag{1.1}$$

where u is the mean velocity of the combustion products, ρ is the density, p is the pressure, S is the cross-sectional area of the chamber, $\varkappa$ is the polytropic index, x is the longitudinal coordinate, t is time, and p_0 and ρ_0 are the pressure and density values in a stationary flow behind the combustion front.

The deformations of the chamber walls are axisymmetric in the case of one-dimensional flow. We make use of moment shell theory to describe the deformations. Considering the inertia forces as well as the structural damping forces, we obtain the following system of equations:

$$\frac{Eh}{1-\nu^2}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\nu}{R}\frac{\partial w}{\partial x}\right)=\rho_c h\frac{\partial^2 v}{\partial t^2}+2\varepsilon_v\rho_c h\frac{\partial v}{\partial t},$$
$$D\frac{\partial^4 w}{\partial x^4}+\frac{Eh}{(1-\nu^2)R}\left(\frac{w}{R}+\nu\frac{\partial v}{\partial x}\right)=p-\rho_c h\frac{\partial^2 w}{\partial t^2}-2\varepsilon_w\rho_c h\frac{\partial w}{\partial t}, \tag{1.2}$$

where w is normal deflection; v is longitudinal displacement, E and ν are the modulus of elasticity and the Poisson ratio, respectively; D is cylindrical rigidity; h is the wall thickness; ρ_c is the density of the shell material; ε_v and ε_w are the damping factors in transverse and longitudinal direction, respectively. The system of equations (1.1), (1.2) will be closed by adding the following relation between S and w:

$$S=\pi(R+w)^2. \tag{1.3}$$

We transform Eqs. (1.1)–(1.3) to dimensionless form, taking the chamber length L, the speed of sound c_0 in stationary flow, the time T required for an acoustic wave to travel from the nozzle to the chamber exit, and the pressure p_0 and density ρ_0 as the characteristic parameters. We introduce the following dimensionless parameters:

$$x_*=\frac{x}{L},\quad \rho_*=\frac{\rho}{\rho_0},\quad p_*=\frac{p}{p_0},\quad t_*=\frac{t}{T},\quad T=\frac{L}{c_0},$$
$$u_*=\frac{u}{c_0},\quad S_*=\frac{S}{\pi R^2},\quad w_*=\frac{w}{R},\quad v_*=\frac{v}{L},\quad c_0=\left(\frac{\varkappa p_0}{\rho_0}\right)^{1/2},$$
$$r_0=\frac{R}{L},\quad \omega^{\circ 2}=\frac{L^2E}{R^2\rho_c c_0^2(1-\nu^2)},\quad \gamma^2=\frac{R^2h^2}{12L^4},$$
$$\varepsilon_{v*}=\frac{L\varepsilon_v}{c_0},\quad \varepsilon_{w*}=\frac{L\varepsilon_w}{c_0},\quad \mu=\frac{\rho_0L^2}{\rho_c Rh}, \tag{1.4}$$

Where ω° is a parameter proportional to the ratio of the partial frequency of a zero-bending-stress shell to the longitudinal oscillation frequency of the combustion products in a closed chamber; the parameter μr_0^2 is the ratio of the mass of the combustion products in the chamber to the mass of the shell; and γ characterizes the ratio of the flexural rigidity to the tensile strength of the shell. After the dimensionless parameters (1.4) are introduced, Eqs. (1.1)–(1.3) take the form (in the following, the asterisks at the dimensionless quantities are omitted)

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}=-\frac{1}{\varkappa}\frac{\partial p}{\partial x},\qquad p=\rho^\varkappa,$$
$$\frac{\partial}{\partial t}(\rho S)+\frac{\partial}{\partial x}(\rho uS)=0,\qquad S=(1+w)^2,$$
$$\frac{\partial^2 v}{\partial t^2}+2\varepsilon_v\frac{\partial v}{\partial t}-\omega^{\circ 2}r_0^2\left(\frac{\partial^2 v}{\partial x^2}+\nu\frac{\partial w}{\partial x}\right)=0,$$
$$\frac{\partial^2 w}{\partial t^2}+2\varepsilon_w\frac{\partial w}{\partial t}+\omega^{\circ 2}w+\omega^{\circ 2}\gamma^2\frac{\partial^4 w}{\partial x^4}+\omega^{\circ 2}\nu\frac{\partial v}{\partial x}=\frac{\mu}{\varkappa}p. \tag{1.5}$$

We differentiate between unperturbed motion, whose parameters do not depend on time, and small perturbations superposed on it. Unperturbed motion is characterized by the parameters

$$\rho=1,\quad p=1,\quad w=w_0=\mu(\omega^{\circ 2}\varkappa)^{-1},\quad v=v_0=0.$$

The dimensionless velocity u in front of the combustion front is small and may be neglected, while behind the combustion front, it is constant and equal to M. The unperturbed state of the shell is free of bending stresses and satisfies the conditions

$$w=w_0,\ \frac{\partial w}{\partial x}=0,\ v=0\quad (x=0;1).$$

Let us derive variational equations for perturbed motion, assuming the disturbances and their derivatives are small. We set

$$\rho=1+\rho',\quad p=1+p',\quad w=w_0+w'\quad v=v'\quad (0\leqslant x\leqslant 1),$$
$$u=u'\quad (0\leqslant x<x_0),\qquad u=M+u'\quad (x_0<x\leqslant 1).$$

By substituting these expressions into Eq. (1.5), after linearization and considering that $w_0\ll 1$, we obtain (the primes at the disturbances are omitted) that

$$\frac{\partial u_k}{\partial t} + M\delta_{k2}\frac{\partial u_k}{\partial x} + \frac{\partial \rho_k}{\partial x} = 0, \qquad p_k = \varkappa\rho_k,$$
$$\frac{\partial \rho_k}{\partial t} + M\delta_{k2}\frac{\partial \rho_k}{\partial x} + \frac{\partial u_k}{\partial x} + 2\frac{\partial w_k}{\partial t} + 2M\delta_{k2}\frac{\partial w_k}{\partial x} = 0,$$
$$\frac{\partial^2 v_k}{\partial t^2} + 2\varepsilon_v\frac{\partial v_k}{\partial t} - \omega^{\circ 2}r_0^2\left(\frac{\partial^2 v_k}{\partial x^2} + \nu\frac{\partial w}{\partial x}\right) = 0, \tag{1.6}$$
$$\frac{\partial^2 w_k}{\partial t^2} + 2\varepsilon_w\frac{\partial w_k}{\partial t} + \omega^{\circ 2}\left(w_k + \gamma^2\frac{\partial^4 w_k}{\partial x^4} + \nu\frac{\partial v_k}{\partial x}\right) = \mu\rho_k,$$

where δ_{kj} is the Kronecker symbol. Subscript $k = 1$ corresponds to the region in front of the combustion front ($0 \le x < x_0$), and subscript $k = 2$ to the region beyond the combustion front ($x_0 < x \le 1$). Let us examine the boundary conditions

$$u_1 = 0, \quad w_1 = \frac{\partial w_1}{\partial x} = v_1 = 0 \quad (x = 0),$$
$$u_2 - MA[\rho_2] = 0, \quad w_2 = \frac{\partial w_2}{\partial x} = v_2 = 0 \quad (x = 1),$$
$$\rho_1 = \rho_2, \ (u_2 - u_1)/M = \beta\varkappa(\rho_1 - \rho_{1\tau}) - \rho_1, \ w_1 = w_2, \ v_1 = v_2 \quad (x = x_0),$$
$$\frac{\partial w_1}{\partial x} = \frac{\partial w_2}{\partial x}, \ \frac{\partial v_1}{\partial x} = \frac{\partial v_2}{\partial x}, \ \frac{\partial^2 w_1}{\partial x^2} = \frac{\partial^2 w_2}{\partial x^2}, \ \frac{\partial^3 w_1}{\partial x^3} = \frac{\partial^3 w_2}{\partial x^3} \quad (x = x_0). \tag{1.7}$$

The first group of these conditions signifies that the edges of the shell are clamped; the velocity perturbations are zero at the nozzle, while at the chamber exit, in the presence of harmonic oscillations, the velocity perturbation is associated with pressure perturbation by acoustic impedance. In (1.7), A is the acoustic admittance operator of the nozzle which, generally speaking, depends both on the nozzle geometry and the reduced frequency. The second group of conditions corresponds to the combustion front. It is assumed that the pressure (density) perturbations and the strains and forces of the perturbed state of the shell are continuous and that the velocity jump is expressed through a retarded functional [3]. The subscript τ indicates that the corresponding value refers to the instant $t - \tau$, where τ is the mean time lag. The constant β is conventionally known as the interaction index.

In this way, the stability analysis of the stationary combustion process is reduced to the determination of conditions for which all solutions of system (1.6) with boundary conditions (1.7) are damped in time. From Eqs. (1.6), it is easy to obtain equations for analyzing the stability of the combustion process in the case in which the chamber casing operates as a zero-bending-stress shell ($\gamma^2\lambda^2 \ll 1$, where λ is the variability coefficient of the stress-strain state), or in which the chamber casing is absolutely rigid ($w \equiv 0, v \equiv 0$). It should be noted that the use of the equations in zero-bending-stress theory is equivalent to the neglect of dynamic edge effects [4].

2. For analyzing the stability of a stationary combustion process, we seek the solution of system (1.6) in the form

$$u_k = U_k(x)e^{st}, \quad \rho_k = G_k(x)e^{st}, \quad w_k = W_k(x)e^{st}, \quad v_k = V_k(x)e^{st} \tag{2.1}$$

where s is a characteristic exponent. The unperturbed motion is stable, provided all the characteristic exponents of the problem lie in the left half-plane. Substitution of (2.1) into (1.6) yields

$$sU_k + M\delta_{k2}U_k' + G_k' = 0, \ sG_k + M\delta_{k2}G_k' + U_k' + 2sW_k + 2M\delta_{k2}W_k' = 0$$
$$(s^2 + 2\varepsilon_v s)V_k - \omega^{\circ 2}r_0^2(V_k'' + \nu W_k') = 0 \tag{2.2}$$
$$(s^2 + 2\varepsilon_w s + \omega^{\circ 2})W_k + \gamma^2\omega^{\circ 2}W_k^{IV} + \omega^{\circ 2}\nu V_k' - \mu G_k = 0$$

where the primes denote differentiation with respect to the x coordinate. The boundary conditions (1.7) reduce to the form

$$U_1 = W_1 = W_1' = V_1 = 0 \quad (x = 0)$$
$$U_2 = \alpha M G_2, \quad W_2 = W_2' = V_2 = 0 \quad (x = 1)$$
$$\chi G_1 - (U_2 - U_1)/M = 0, \quad G_1 = G_2, \quad V_1 = V_2 \ (x = x_0)$$
$$W_1 = W_2, \ W_1' = W_2', \quad W_1'' = W_2'', \quad W_1''' = W_2''' \ (x = x_0) \tag{2.3}$$
$$\chi = \varkappa\beta(1 - e^{-s\tau}) - 1, \quad \alpha = e^{-i\omega t}A[e^{i\omega t}] \tag{2.4}$$

where α is the acoustic admittance of the nozzle.

The solution of system (2.2) can be represented in the form

$$U_k = \sum_{j=1}^{8} A_{kj} \exp(l_{kj}x), \quad G_k = \sum_{j=1}^{8} B_{kj} \exp(l_{kj}x),$$
$$W_k = \sum_{j=1}^{8} C_{kj} \exp(l_{kj}x), \quad V_k = \sum_{j=1}^{8} D_{kj} \exp(l_{kj}x) \quad (k = 1, 2). \tag{2.5}$$

The characteristic exponents l_{kj} are determined from the equations

$$\det \| a_{\lambda\mu}^{(k)} \| = 0 \quad (k = 1, 2)\ (\lambda, \mu = 1, \ldots, 4). \tag{2.6}$$

In these equations, the nonvanishing terms are

$$a_{11}^{(k)} = a_{22}^{(k)} = s + M\delta_{k2} l_k, \quad a_{12}^{(k)} = a_{21}^{(k)} = l_k,$$
$$a_{23}^{(k)} = 2a_{11}^{(k)}, \quad a_{32}^{(k)} = -\mu, \quad a_{33}^{(k)} = s^2 + 2\varepsilon_w s + \omega^{\circ 2} + \gamma^2\omega^{\circ 2} l_k^4,$$
$$a_{34}^{(k)} = \omega^{\circ 2}\nu l_k, \quad a_{43}^{(k)} = -r_0^2\omega^{\circ 2}\nu l_k, \quad a_{44}^{(k)} = s^2 + 2\varepsilon_v s - r_0^2\omega^{\circ 2} l_k^2.$$

Equations (2.6) are eighth-order algebraic equations. The constants of integration A, B, C, and D may be expressed through 16 constants N_{kj} ($j = 1, \ldots, 8$; $k = 1, 2$). By satisfying the boundary conditions (2.3), we arrive at a system of 16 homogeneous linear equations for determining the constants N_{kj}. The existence of nonzero solutions for N_{kj} requires a zero determinant of the system. From here, we obtain an equation for determining the characteristic exponents s. Due to its extreme awkwardness, we do not present it here. A special case of this equation is presented in the following.

Convenient parameters for characterizing the combustion process are mean time lag τ and the interaction index β. Therefore, in order to determine the instability region, we map the imaginary axis of the plane of the complex variable onto the plane of the real variables τ, β. To this end, we solve the equation for the characteristic exponents with respect to the parameter χ. Setting $s = i\omega$, and separating the imaginary and real parts with allowance for (2.5), we obtain

$$\beta\varkappa(1 - \cos\omega\tau) = f, \quad \beta\varkappa \sin\omega\tau = \varphi \tag{2.7}$$

where $f - 1$ and φ are the real and imaginary parts of the solution of the characteristic equation with respect to χ, after the substitution of $s = i\omega$.

3. The expressions for f and φ in the general case are very cumbersome. For $\nu = 0$, the equations for the longitudinal and transverse displacements of the chamber walls separate. In this case, the longitudinal vibrations of the chamber walls may be neglected, since they have no influence on the combustion process.

For a sufficiently long chamber, it may be assumed that the combustion front is located at the nozzle ($x_0 = 0$). Finally, the length of the supersonic portion of the nozzle is much smaller than the chamber length, the expression $\alpha = (\varkappa - 1)/2$ is applicable for the acoustic admittance of the nozzle. For these assumptions, the characteristic equations for the exponents s, solved with respect to the parameter χ, has the form

$$\chi = \frac{1}{M}\frac{\Delta_1}{\Delta_2}, \tag{3.1}$$

$$\Delta_1 = \begin{vmatrix} 1 & \cdots & 1 \\ l_1 & \cdots & l_6 \\ e^{l_1} & \cdots & e^{l_6} \\ l_1 e^{l_1} & \cdots & l_6 e^{l_6} \\ (a_1 - \alpha M b_1)e^{l_1} & \cdots & (a_6 - \alpha M b_6)e^{l_6} \\ a_1 & \cdots & a_6 \end{vmatrix}, \tag{3.2}$$

$$a_j = -\frac{l_j}{(s + Ml_j)\mu}(\gamma^2\omega^{\circ 2} l_j^4 + S_1), \qquad b_j = -\frac{1}{\mu}(\gamma^2\omega^{\circ 2} l_j^4 + S_1),$$

$$S_1 = s^2 + 2\varepsilon_w s + \omega^{\circ 2}.$$

The determinant Δ_2 is obtained from Δ_1 by substituting b for a in the last row with the corresponding subscripts. The characteristic exponents l_j are determined from the equation

$$\begin{gathered}\gamma^2 (m_6 l^6 + m_5 l^5 + m_4 l^4) + m_3 l^3 + m_2 l^2 + m_1 l + m_0 = 0, \\ m_6 = \omega^{\circ 2} (M^2 - 1), \quad m_5 = 2\omega^{\circ 2} M s, \\ m_4 = \omega^{\circ 2} s^2, \quad m_3 = 0, \\ m_2 = S_1 (M^2 - 1) + 2\mu M^2, \quad m_1 = 2Ms (S_1 + 2\mu), \\ m_0 = -\omega^2 (S_1 + 2\mu).\end{gathered} \tag{3.3}$$

The expressions for f and φ can be obtained after substitution of $s = i\omega$ into (3.2) and (3.3) and separation of the real and imaginary parts in (3.1).

As another special case, we examine the problem of combustion stability under the assumption that the deformations of the chamber walls are described by zero-bending-stress shell theory. In this case, for (3.1) we obtain

$$\begin{gathered}\chi = \frac{\Phi}{M}(1 + \operatorname{th}\Phi i\omega) - \frac{2\Phi(\Phi - \alpha M)}{M[(\Phi + \alpha M)\, e^{(l_1 - l_2)(1 - x_0)} + (\Phi - \alpha M)]}, \\ \Phi^2 = 1 + \frac{2\mu\varkappa}{\omega^{\circ 2} + 2\varepsilon i\omega - \omega^2}, \\ l_1 = \frac{i\Phi\omega}{1 - M\Phi}, \quad l_2 = -\frac{i\Phi\omega}{1 + M\Phi}.\end{gathered} \tag{3.4}$$

Separating the real and imaginary parts in (3.4) with allowance for $\alpha = \alpha_r + i\alpha_i$ and $\Phi = \Phi_r + i\Phi_i$, we obtain for f and φ an explicit expression which, owing to its awkwardness, will be omitted here.

The formulas simplify somewhat if one assumes that the combustion front is located at the nozzle ($x_0 = 0$) and that the acoustic admittance of the nozzle is equal to $(\varkappa - 1)/2$.

4. Relations (2.7) define in parametric form, in the τ, β plane, a certain curve $F(\tau, \beta) = 0$ which is the image of the imaginary axis in the plane of the given parameters. The mapping of the right half-plane of the complex variable onto the τ, β plane has the form of a multisheeted surface.

Therefore, in addition to computational difficulties, the use of (2.7) also involves certain major difficulties.

First of all, only certain segments of the curve $F(\tau, \beta) = 0$ correspond to the boundaries of the instability region. Secondly, to each sheet of the surface, there corresponds apparently a specific type of instability. A classification of the sheets in terms of instability types is imperative for a physical interpretation of the results and for their application in practice.

The segments of the curve $F(\tau, \beta) = 0$ which correspond to the boundary of the instability region can be identified on the basis of the following considerations. It is not difficult to establish that for $\beta = 0$ all the characteristic exponents have negative real parts. Consequently, unperturbed motion will remain stable also in a certain sufficiently narrow strip $0 \le \beta < \beta_0(\tau)$. The value of $\beta_0(\tau)$ is determined by moving from the axis $\beta = 0$ toward the positive values of β until the first intersection with the curve $F(\tau, \beta) = 0$, defined by relation (2.7), is reached.

The multisheeted nature of the surface, which is the image of the right half-plane of the variable s, is of dual origin. First, it is generated owing to the presence of an infinite denumerable set of the natural frequencies of the acoustoelastic system. Second, the multisheeted nature of the surface is associated with the phenomenon whereby an instability observed for the time lag τ repeats itself at other times which, roughly speaking, differ from τ by a multiple of the natural-oscillation periods. By solving (2.7) with respect to β and τ, we obtain

$$\beta = \frac{f^2 + \varphi^2}{2\varkappa f}, \tag{4.1}$$

$$\tau = \frac{1}{\omega}\left\{(2n_t + 1)\pi - \operatorname{sign}\varphi \arccos\left(\frac{f}{\beta\varphi} - 1\right)\right\} \quad (n_t = 0, 1, 2, \ldots). \tag{4.2}$$

where n_t is the number of complete periods of oscillation which "fit into" the variable portion of the time lag.

From formula (4.1) and the expressions for f and φ it can be seen that the interaction index is a unique fluctuating function of frequency ω. In the case of an absolutely rigid chamber ($w \equiv 0, v \equiv 0$), there exists an unnumerable multitude of minima near the frequencies $\omega = n_x\pi$, where $n_x = 0, 1, 2, \ldots$ (Fig. 1). The number n_x characterizes the mode of oscillation—the number of nodes along length L.

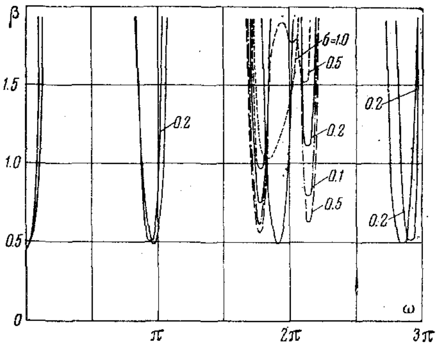

Fig. 1

In Fig. 1, the solid curves correspond to $\beta(\omega)$ for this case.

The picture is more complex in the case of elastic walls. The pliability of the walls, however, has an effect only near frequencies ω that are close to the partial frequencies of the natural vibrations of the shell.

The frequencies of coupled acoustoelastic oscillations are situated in the neighborhood of the natural partial frequency of the shell. The presence of these coupled oscillations leads to the appearance of additional minima on the $\beta(\omega)$ curve. The number of the excited acoustoelastic form-shapes of oscillations depends strongly on the magnitude of structural damping and on the ratio of the partial frequencies of the shell to those of the combustion products.

It has been said above, that the second reason for the multisheeted nature of the instability region is the ambiguity of function $\tau(\omega)$ defined by (4.2). In order to construct the instability regions in the τ, β plane, it is necessary to use formulas (4.1), (4.2). Each sheet of the instability region is assigned two indices, n_x and n_t, one of which characterizes the form shape of the oscillations and the other the number of oscillation periods contained in the time τ.

5. Let us dwell on a discussion of numerical results. Calculations from formulas (3.4), (4.1), (4.2) were performed for a zero-bending-stress shell, making use of the following data: $x_0 = 0$, $\varkappa = 1.2$, $M = 0.213$, $\alpha = 0.1$, $\mu = 0.5$, $\omega^\circ = 2\pi$; $3\pi/2$, $\delta = 2\pi\varepsilon/\omega^\circ = 0\text{–}1$. The results of the calculations are shown in Figs. 1–3. Figure 1 shows plots of $\beta(\omega)$ for $\omega^\circ = 2\pi$ and various values of the decrement δ. The curves show that in the case of an elastic chamber, there appear additional loops that correspond to coupled acoustoelastic oscillations. The loops situated at some distance from ω° hardly change their shape. The most pronounced change in the relationship of the interaction index $\beta(\omega)$ occurs in the case in which the partial frequencies of the natural vibrations of an elastic shell coincide with the natural oscillations of combustion products in a closed chamber. In this case, there is a maximum difference between the coupled acoustoelastic frequencies and the partial frequencies.

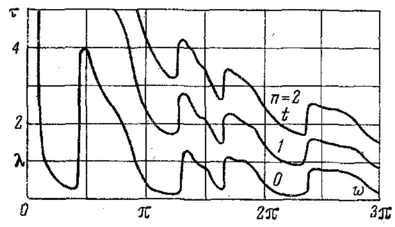

Fig. 2

The influence of the elasticity of the chamber walls is found to be less pronounced when the partial frequency of the shell lies between the acoustic frequencies. Introduction of strong structural damping has the effect of almost eliminating the influence of chamber-wall elasticity on the combustion process.

It is noteworthy that $\omega_f = n_x\pi$ in dimensional values has the form $c/R = n_x\pi c_0/L$, where c is the propagation rate of longitudinal waves in the wall material. For moderate values of n_x, this condition does not hold. Thus, the problem studied is of a methodical nature.

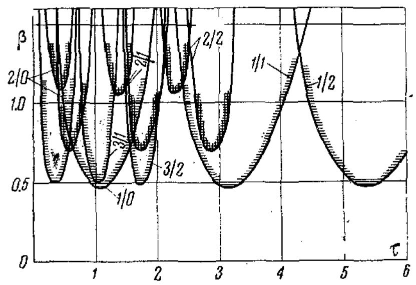

Fig. 3

Figure 2 shows a characteristic plot of the time lag $\tau(\omega)$ for $\omega^\circ = 3\pi/2$ and $\delta = 0.2$. The instability regions in the β, τ plane are shown in Fig. 3 for $\omega^\circ = 2$ and $\delta = 0.2$. The instability region is situated in the proximity of $\beta = 0$. Each sheet of the instability region is assigned two values, n_x and n_t, which characterize the form-shape of the excited oscillations and the ratio of the oscillation period to the time lag, respectively, (in Figs. 3 and 4, the values of n_x are given in the numerator and those of n_t in the denominator). The sheets of the instability region for which ω is close to ω° remain almost the same as in the case of rigid chamber walls. In Fig. 3, the sheets which correspond to the coupled acoustoelastic oscillations are cross-hatched.

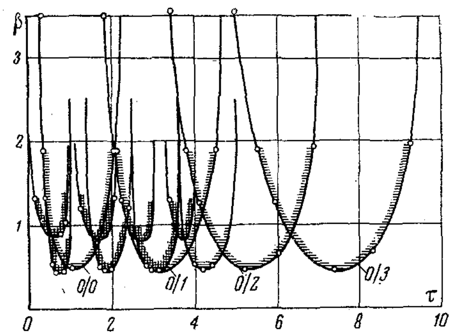

Fig. 4

In the case in which the deformations of the shell are described by moment theory, calculations were performed for $\nu = 0$, $x_0 = 0$, $\alpha = (\varkappa - 1)/2$. Calculations on the basis of formulas (3.1)–(3.3), (4.1), and (4.2) were programmed for the Minsk-22 computer. Complex values for the coefficients m_j of the characteristic equation (3.3) were obtained by introducing numerical values for ω°, γ, ω, and other parameters. After finding the roots of the characteristic equation, it was possible to calculate the values of the coefficients a_j, b_j and of the determinants Δ_1 and Δ_2. By separating the real from the imaginary parts, we calculated f and φ which, in turn, were used to calculate the interaction index β and the time lag τ from formulas (4.1) and (4.2). After determining β and τ, ω was varied and the cycle was repeated up to a certain prescribed value ω_{max}. Calculations were repeated for other values of γ and, finally, for another value of ω°.

In this case, a distinctive feature of the calculations was performing operations with complex numbers at each step with the aid of standard subroutines for operations with complex numbers.

Figure 4 shows the points calculated in the instability region in the β, τ plane for $\nu = 0$, $x_0 = 0$, $\gamma = 0.05$, $\omega^\circ = 2\pi$, $\delta = 0.2$. The classification of the instability regions proved more difficult than in the case of a zero-bending-stress shell. This applies primarily to the characteristic of the form-shapes of oscillations, where dynamic edge effects play a significant role [4]. In Fig. 4, the regions which correspond to the excitation of coupled acoustoelastic oscillations are cross-hatched.

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